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MATHEMATICAL MODELING OF THERMOMECHANICAL PROCESSES IN AIRCRAFT STRUCTURES

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Consideration is given to some thermomechanical models of the behavior of structural materials that are based on the thermodynamics of irreversible processes and on the analysis of physical processes in polycrystalline materials.

Many important structural elements of aircraft operate under intense thermal and mechanical effects. The scientific foundation of analyzing the efficiency and durability of these thermally stressed structures is thermomechanics which is based on the thermodynamics of irreversible processes, thermophysics, the mechanics of a deformable rigid body, and other fundamental disciplines, and widely uses the methods of applied mathematics and state-of-the-art computing aids. In this sense, thermomechanics is a characteristic example of the synthesis of new scientific trends brought about by the needs of engineering practice.

The value and reliability of the results of mathematical modeling of thermomechanical processes are essentially dependent on the adequacy of the models of the behavior of structural materials at variable temperatures and loadings, on the completeness of allowance for the actual conditions of thermal and force loading of structural elements in technological, testing, and operating modes, and on the perfection and efficiency of the applied computing methods and aids.

One of the possible variants of constructing a thermomechanical model of the behavior of structural materials is based on the model of a medium with internal state parameters. Such a medium is described using one scalar parameter, two vector parameters, and one tensor parameter. Their physical meaning is as follows. Equilibrium thermodynamic processes are characterized by the use of the absolute temperature $T(x_1, x_2, x_3, t)$, which determines the free energy of the system. However, if the process in the thermodynamic system is nonequilibrium (or locally nonequilibrium), consideration is also given to the thermodynamic temperature $\chi(x_1, x_2, x_3, t)$ (the scalar internal parameter), which coincides with the absolute temperature, if its rate of change is equal to zero [1]. The absolute temperature T is a measure of the mean kinetic energy of molecules in the equilibrium process, and the thermodynamic temperature, in the nonequilibrium process. The propagation of heat is characterized by two internal vector parameters $\chi_i^{(1)}(x_1, x_2, x_3, t)$ and $\chi_i^{(2)}(x_1, x_2, x_3, t)$, which, for example, for crystalline materials can be associated with the vectors of distribution density of phonons and electrons, respectively [2]. The internal tensor parameter $\chi_{ij}(x_1, x_2, x_3, t)$ represents the nonequilibrium mechanical processes at the microlevel.

The kinetic equations for changes in χ, $\chi_i^{(1)}$, $\chi_i^{(2)}$, and χ_{ij} in a linear approximation are taken in the form [3, 4]

$$\tau_T \dot{\chi} = \bar{\chi} - \chi \,, \quad \tau_q^{(1)} \dot{\chi}_i^{(1)} = \bar{\chi}_i^{(1)} - \chi_i^{(1)} \,, \quad \tau_q^{(2)} \dot{\chi}_i^{(2)} = \bar{\chi}_i^{(2)} - \chi_i^{(2)} \,, \quad \tau_\sigma \dot{\chi}_{ij} = \bar{\chi}_{ij} - \chi_{ij} \,. \tag{1}$$

The expression for the free energy is specified in a quadratic form

$$\rho A\, (\varepsilon_{kl}, T, \chi, \chi_k^{(1)}, \chi_k^{(2)}, \chi_{kl}) = C_{ijkl}\, (\varepsilon_{kl} - \varepsilon_{kl}^{(T)})\, (\varepsilon_{ij} - \varepsilon_{ij}^{(T)})/2 -$$

$$- D_{ijkl}\, \beta_{kl}\, (\varepsilon_{ij} - \varepsilon_{ij}^{(T)}) - H_{ijkl}\, \chi_{kl}\, (\varepsilon_{ij} - \varepsilon_{ij}^{(T)}) + F_{ij}^{(1)}\, \chi_i^{(1)}\, \chi_j^{(1)}/2 +$$

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$$+F_{ij}^{(2)}\chi_j^{(2)}\chi_i^{(2)}/2+K_{ijkl}\chi_{kl}\chi_{ij}/2+\rho B(T)+\rho B_1(T,\chi)-$$

$$-C_{ijkl}(-\varepsilon_{kl}^{(T)})(-\varepsilon_{ij}^{(T)})/2+D_{ijkl}\beta_{kl}(-\varepsilon_{ij}^{(T)})\,, \tag{2}$$

$$A(0,T_0,T_0,0,0,0)=0\,.$$

Equation (2) was written assuming that total and thermal deformations are insignificant: $\|\varepsilon_{ij}\| << 1$, $\|\varepsilon_{ij}^{(T)}\| << 1$, $\|\varepsilon_{ij}^{(\chi)}\| << 1$ and $\|\chi_j^{(1)}\| << 1$, $\|\chi_{ij}^{(2)}\| << 1$, $\|\chi_{ij}\| << 1$, where $\|\cdot\|$ is the Euclidean matrix norm, and $\beta_{ij} = \varepsilon_{ij}^{\chi} - \varepsilon_{ij}^{(T)}$, $i, j, k, l = 1, 2, 3$. In Eq. (2) and thereafter, we agreed on summation with respect to the double Latin indices. Clearly, if $\chi \to T$, then $\beta_{ij} \to 0$. Assuming a linear dependence of $\varepsilon_{ij}^{(T)}$ on T and of $\varepsilon_{ij}^{(\chi)}$ on χ, we set $\partial\varepsilon_{ij}^{(\chi)}/\partial\chi = \partial\varepsilon_{ij}^{(T)}/\partial T = \alpha_{ij}^{(T)}$ and $B_0(T_0) = 0$, $B_1(T_0, T_0) = 0$. In Eq. (2), the independence of the free energy of T and χ gradients, i.e., $\partial A/\partial(\partial T/\partial x_k) = 0$ and $\partial A/\partial(\partial\chi/\partial x_k) = 0$, follows from a simultaneous consideration of the first and second (in the form of the Clausius–Duhem equation) laws of thermodynamics.

The components of the stress tensor σ_{ij} and the entropy h are related to the free energy as $\sigma_{ij} = \rho\partial A/\partial\varepsilon_{ij}$ and $h = -\partial A/\partial T$, i.e., allowing for Eq. (2)

$$\sigma_{ij}=C_{ijkl}(\varepsilon_{kl}-\varepsilon_{kl}^{(T)})-D_{ijkl}\beta_{kl}-H_{ijkl}\chi_{kl}\,, \tag{3}$$

$$h=\alpha_{ij}^{(T)}(C_{ijkl}\varepsilon_{kl}-H_{ijkl}\chi_{kl})/\rho-\partial B_0/\partial T-\partial B_1/\partial T\,, \tag{4}$$

which makes it possible to write the energy equation in the form

$$-\rho T\partial^2 B_0/\partial T^2\dot{T}-\rho T\partial^2 B_1/(\partial T\partial\chi)\,\dot{\chi}=$$

$$=-T\alpha_{ij}^{(T)}(C_{ijkl}\dot{\varepsilon}_{kl}-H_{ijkl}\dot{\chi}_{kl})-\partial q_i/\partial x_i+\rho r\,. \tag{5}$$

The subsequent concretization of Eqs. (3) and (5) involves a selection of the form of the functions of steady-state values for the internal state parameters and the vector of the heat flux density with the components $q_i(x_1, x_2, x_3, t)$. In the simplest case, we set

$$\begin{gathered}\bar{\chi}=T\,,\quad \bar{\chi}_i^{(1)}=-Z_{ij}^{(1)}\partial T/\partial x_j-Z_{ij}^{(2)}\partial\chi/\partial x_j\,,\\ \bar{\chi}_i^{(2)}=-Y_{ij}^{(1)}\partial T/\partial x_j-Y_{ij}^{(2)}\partial\chi/\partial x_j\,,\\ \bar{\chi}_{ij}=X_{ijkl}\varepsilon_{kl}\,,\quad q_i=\varphi_{ij}^{(1)}\chi_j^{(1)}+\varphi_{ij}^{(2)}\chi_j^{(2)}\,,\end{gathered} \tag{6}$$

which is consistent with the fundamental principles of rational thermodynamics [1]. Then, the heat conduction is described by the equation arising from Eqs. (5), (2), and (6)

$$\rho c_\varepsilon\dot{T}+\rho c_\varepsilon'/\tau_T\int_0^t\exp\left(-\frac{t-t'}{\tau_T}\right)\frac{\partial T}{\partial t'}dt'+T\alpha_{ij}^{(T)}\left[C_{ijkl}\dot{\varepsilon}_{kl}-R_{ijkl}\int_0^t\exp\left(-\frac{t-t'}{\tau_\sigma}\right)\frac{\partial\varepsilon_{kl}}{\partial t'}dt'/\tau_\sigma\right]=$$

$$=\frac{\partial}{\partial x_i}\left\{\lambda_{ij}^{(1)}\left[\frac{\partial T}{\partial x_j}-\int_0^t\exp\left(-\frac{t-t'}{\tau_q^{(1)}}\right)\frac{\partial^2 T}{\partial t'\partial x_j}dt'\right]+\lambda_{ij}^{(2)}\left[\frac{\partial T}{\partial x_j}-\int_0^t\exp\left(-\frac{t-t'}{\tau_T}\right)\frac{\partial^2 T}{\partial t'\partial x_j}dt'-\right.\right.$$

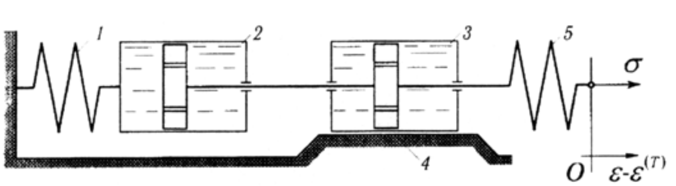

Fig. 1. Mechanical analog of the glide system in a crystal grain of a polycrystalline material.

$$-\int_0^t \exp\left(\frac{t-t'}{\tau_q^{(1)}}\right)\frac{1}{\tau_T}\int_0^{t'} \exp\left(-\frac{t-t''}{\tau_T}\right)\frac{\partial^2 T}{\partial t''\partial x_j}\,dt''dt'\Bigg] + \lambda_{ij}^{(3)}\left[\frac{\partial T}{\partial x_j} - \int_0^t \exp\left(-\frac{t-t'}{\tau_q^{(2)}}\right)\frac{\partial^2 T}{\partial t'\partial x_j}\,dt'\right] +$$

$$+\lambda_{ij}^{(4)}\left[\frac{\partial T}{\partial x_j} - \int_0^t \exp\left(-\frac{t-t'}{\tau_T}\right)\frac{\partial^2 T}{\partial t'\partial x_j}\,dt' - \frac{1}{\tau_T}\int_0^t \exp\left(-\frac{t-t'}{\tau_q^{(2)}}\right) \times \int_0^{t'} \exp\left(-\frac{t-t''}{\tau_T}\right)\frac{\partial^2 T}{\partial t''\partial x_j}\,dt''dt'\right]\Bigg\} + \rho r. \tag{7}$$

A concretization of Eq. (3) with allowance for the assumptions made has the form

$$\sigma_{ij} = C_{ijkl}\,(\varepsilon_{kl} - \varepsilon_{kl}^{(T)}) + D_{ijkl}\,\alpha_{kl}^{(T)}\int_0^t \exp\left(-\frac{t-t'}{\tau_T}\right)\frac{\partial T}{\partial t'}\,dt' - R_{ijkl}\left[\varepsilon_{kl} - \int_0^t \exp\left(-\frac{t-t'}{\tau_\sigma}\right)\frac{\partial \varepsilon_{kl}}{\partial t'}\,dt'\right]. \tag{8}$$

Relations (7) and (8) describe a fairly wide class of relaxing media and account for the following factors: delay of the deformation and heat accumulation in high-intensity loading (the second terms on the left-hand side of Eq. (7) and on the right-hand side of Eq. (8)), finiteness of the velocities of heat propagation V_1 and V_2 for each physical process of heat conduction, which are defined by the relations

$$\rho c_\varepsilon V_1^2 - \lambda_{ij}^{(1)}\, n_j n_i/\tau_q^{(1)} = 0\,, \quad \rho c_\varepsilon V_2^2 - \lambda_{ij}^{(3)}\, n_j n_i/\tau_q^{(2)} = 0\,;$$

the effect of the nonequilibrium of the heat accumulation on the heat conduction (the second and fourth terms in the square brackets on the right-hand side of Eq. (7)), and also the effects of thermodynamic connectedness (the third term on the left-hand side of Eq. (7)).

From relations (7) and (8) it is possible to obtain their asymptotic representations for $\tau \to 0$ ($\tau = \tau_T$, $\tau_q^{(1)}$, $\tau_q^{(2)}$, τ_σ). In these cases,

$$\lim_{\tau\to 0}\int_0^t \exp\left(-\frac{t-t'}{\tau}\right)\frac{\partial(\cdot)}{\partial t'}\,dt' = 0\,, \quad \lim_{\tau\to 0}\frac{1}{\tau}\int_0^t \exp\left(-\frac{t-t'}{\tau}\right)\frac{\partial(\cdot)}{\partial t'}\,dt' = \frac{\partial(\cdot)}{\partial t}$$

and the asymptotic representations following from relations (7) and (8) describe particular thermomechanical models of relaxing media. For example, for $\tau_T \to 0$, $\tau_q^{(1)} \to 0$, and $\tau_q^{(2)} \to 0$, relations (7) and (8) describe a standand linear medium with allowance for the effect of the connectedness of the deformation and heat conduction.

In some cases, practical calculations of thermally stressed aircraft structures necessitate a model of a structural material that accounts not only for the connection between the deformation and the stresses in the form of (3) but also for inelastic deformation under nonisothermal conditions. Let us consider one of the ways to construct this model taking uniaxial loading as a simple case in point. As a prototype, use will be made of the mechanical analog of the glide system in a crystal grain of polycrystalline material [5], depicted in Fig. 1.

At relatively low temperatures, thermally activated processes in a structural material proceed rather slowly (the viscosity of the fluid in the viscous-friction elements 2 and 3 in the figure is high). Then, the increment $d\varepsilon^{(p)}$ in the instantaneous plastic deformation occurs under the condition

$$|\sigma - \sigma'| = \sigma^* , \tag{9}$$

where σ and σ' in the figure correspond to the external force and tension of spring 1, which in the general case has a nonlinear characteristic, and σ^* in the mechanical analog corresponds to the resistance force due to dry friction between element 3 and base 4 (spring 5 in the figure models the property of linear elasticity of the material).

We assume that, under isothermal conditions, σ' is a function of $\varepsilon^{(p)}$ and the temperature T

$$\sigma' = f'(\varepsilon^{(p)}, T) , \tag{10}$$

and σ^* is a function of T and the absolute accumulated plastic deformation

$$\sigma^* = f^*(q_p^*, T) , \quad q_p^* = \int\limits_0^{\varepsilon^{(p)}} |d\varepsilon^{(p)}| . \tag{11}$$

Condition (9) is necessary but not sufficient for the occurrence of increments in the instantaneous plastic deformation. It should be completed with the condition

$$d' \, |\sigma - \sigma'| = d' \, ((\sigma - \sigma') \operatorname{sign} (\sigma - \sigma')) > d'\sigma^* , \tag{12}$$

where a prime on the differential sign indicates that the increments are calculated disregarding the material hardening caused by the running plastic deformation, i.e., at $d\varepsilon^{(p)} = 0$. Then, with account for Eqs. (10) and (11) in lieu of Eq. (12), we obtain

$$(d\sigma - k_T dT) \operatorname{sign} (\sigma - \sigma') > k_T^* dT , \quad k_T = \frac{\partial f'}{\partial T} , \quad k_T^* = \frac{\partial f^*}{\partial T} . \tag{13}$$

With condition (12) satisfied, the conversion, in expression (13), from the inequality to the equality will correspond to a neutral loading, and the replacement of the "greater" sign with the "less" sign, to the initiation of elastic unloading of the material. In these two cases, there is no increment in the instantaneous plastic deformation, i.e., $d\varepsilon^{(p)} = 0$.

Since condition (9) is a necessary condition of the plastic deformation of the material, according to Eqs. (10) and (11), the following equality holds true for total increments:

$$(d\sigma - k_T dT - k_p d\varepsilon^{(p)}) \operatorname{sign} (\sigma - \sigma') = k_T^* dT + k_p^* dq_p^* , \quad k_p^* = \frac{\partial f^*}{\partial q_p^*} . \tag{14}$$

Comparing Eqs. (13) and (14), we obtain

$$k_p d\varepsilon^{(p)} \operatorname{sign} (\sigma - \sigma') + k_p^* dq_p^* > 0 , \quad k_p = \frac{\partial f'}{\partial \varepsilon^{(p)}} .$$

For steadily deformable materials, the sign of the increment $d\varepsilon^{(p)}$ coincides with the sign of the difference $\sigma - \sigma'$, i.e., with account for (11) we have

$$d\varepsilon^{(p)} \operatorname{sign} (\sigma - \sigma') = |d\varepsilon^{(p)}| = dq_p^* .$$

From the last two relations for hardening coefficients we obtain a constraint of the form

$$k_p + k_p^* = \frac{\partial f'}{\partial \varepsilon^{(p)}} + \frac{\partial f^*}{\partial q_p^*} > 0 . \tag{15}$$

In the particular case of a material with isotropic softening ($k_p^* < 0$), a steady plastic deformation is possible, if $k_p > |k_p^*|$. In the limit case $k_p = |k_p^*|$, such as for an ideal plastic material with a constant yield strength, a unique connection of the increments $d\sigma$ and $d\varepsilon^{(p)}$ can be established only given additional conditions.

A temperature rise intensifies thermally activated processes in the material, and even under time (t)–invariable conditions of thermal and mechanical effects, an increment in the inelastic deformation appears as a result of the creep effect. For the rate $\dot{\varepsilon}^{(c)}$ of the creep $\varepsilon^{(c)}$, we adopt the relation

$$\dot{\varepsilon}^{(c)} = \frac{\partial \varepsilon^{(c)}}{\partial t} = f_c\,(\sigma - \sigma', T) .$$

In the mechanical analog (see the figure), the finite viscosity of the fluid in the nonlinear viscous-friction elements 2 and 3 corresponds to the manifestation of the creep effect, with the characteristic of element 3 corresponding to the function f_c. The thermal softening of the material causes a σ' decrease in magnitude such that under nonisothermal conditions

$$\dot{\sigma}' = k_T \dot{T} + k_p \dot{\varepsilon}^{(p)} + k_c f_c\,(\sigma - \sigma', T) - f''\,(\sigma', T) , \quad k_c = \frac{\partial f'}{\partial \varepsilon^{(c)}} . \tag{16}$$

Now, with the manifestation of the creep effect, $f' = f'\,(\varepsilon^{(p)}, \varepsilon^{(c)}, T)$ and the function f'' corresponds to the characteristic of the viscous-friction element 2 in the mechanical analog (see the figure). In a particular case with the same mechanism of anisotropic hardnening of the material due to the instantaneous plastic deformation and creep we have $k_p = k_c$.

The rate of total deformation

$$\dot{\varepsilon} = \dot{\varepsilon}^{(e)} + \dot{\varepsilon}^{(T)} + \dot{\varepsilon}^{(p)} + \dot{\varepsilon}^{(c)}$$

includes the rate

$$\dot{\varepsilon}^{(e)} = \frac{\dot{\sigma}}{E} - \frac{\sigma}{E^2} \frac{dE}{dT} \dot{T}$$

of the elastic deformation $\varepsilon^{(e)}$, where $\dot{T}$ is the rate of change in the temperature, and also the rates $\dot{\varepsilon}^{(T)}$, $\dot{\varepsilon}^{(p)}$, and $\dot{\varepsilon}^{(c)}$ of the thermal and instantaneous plastic deformation and creep, respectively. In the mechanical analog (see the figure), the elasticity modulus corresponds to the rigidity of spring 5, $\dot{\varepsilon}^{(p)}$ is proportional to the velocity of the element 3 movement relative to base 4, and $\dot{\varepsilon} - \dot{\varepsilon}^{(T)}$ are the velocities of the point of application of the external force.

At a high temperature, the thermal softening of the material also has an effect on σ^*. With fairly long holding of the material under isothermal conditions ($T = 0$) without inelastic deformation ($\dot{\varepsilon}^{(p)} = \dot{\varepsilon}^{(c)} = 0$), the value of σ^* should tend to the level $\sigma^*(T)$, characteristic of the given material, which corresponds to the yield strength after high-temperature annealing. Let

$$\dot{\sigma}^* = k_T^* \dot{T} + k_p^* \dot{q}_p^* + k_c^* \dot{q}_c^* - f^{**}\,(\sigma^* - \sigma_0^*) , \quad k_c^* = \frac{\partial f^*}{\partial q_c^*} , \quad q_c^* = \int_0^t |\dot{\varepsilon}^{(c)}|\, dt . \tag{17}$$

Here, in the general case, $f^* = f^*(q_p^*, q_c^*, T)$, and the function f^{**} characterizes the rate of removal of the isotropic hardening and represents the effect of time delay of the variation in the material yield strength relative to the temperature change [5]. In the particular case with the same mechanisms of isotropic hardening of the material owing to accumulated values of the instantaneous plastic deformation q_p^* and the creep q_c^* we have $k_p^* = k_c^*$. Now, in addition to prerequisite (9) for the origination of instantaneous plastic deformations, according to Eqs. (12), (16), and (17), in lieu of Eq. (13) we obtain

$$(\dot{\sigma} - k_T\dot{T} - k_c\,\dot{\varepsilon}^{(c)} + f''(\sigma', T))\,\mathrm{sign}\,(\sigma - \sigma') > k_T^*\dot{T} + k_c^*\,\dot{q}_c^* - f^{**}(\sigma^* - \sigma_0^*)\,,$$

but constraint (15) holds.

Thus, the considered variant of the material model describes basic effects characteristic of the inelastic behavior of a structural material under nonisothermal conditions. Among these effects the following should be noted: variation in the yield strength with change in the deformation direction (the Bauschinger effect); conformity to the Masing principle extended to nonisothermal conditions [5]; cyclic isotropic hardening and softening of the material; unsteady and steady stages of creep at constant load; interference of creep and instantaneous plastic deformation; variation in the creep rate with same-sign stepwise loading and alternating-sign loading; reverse creep during the unloading and in the unloaded state; relaxation of microstresses and restoration of plastic properties (relaxation) of the material; recrystallization effect on the removal of the isotropic hardening; and delay of variation in the yield strength under nonisothermal conditions.

Notwithstanding the long list of the considered effects, the selection of parameters for the examined model variant requires a relatively small volume of experimental data obtained in standard tests of the material specimens in uniaxial loading. Physical concepts of the micromechanism of the inelastic deformation of polycrystalline materials permit the concretization of the form of the functions f_c, f'', and f^{**} and thus the simplification of selecting numerical values for the model parameters. These functions can be represented by the product of two multipliers, one of which depends only on temperature and the other is expressed by a hyperbolic sine of an argument, which, in turn, depends on relevant stress and temperature. Then, it is possible to write

$$f_c = C(T)\sinh B(T)\,\frac{\sigma - \sigma'}{\sigma^*}\,,$$

$$f'' = C'(T)\sinh B'(T)\,\frac{\sigma'}{\sigma^*}\,,\quad f^{**} = C^*(T)\sinh B^*(T)\,\frac{\sigma^* - \sigma_0^*}{\sigma^*}\,.$$

The number of the selected model parameters decreases under the assumption that the effects of the instantaneous plastic deformation and creep on the material hardening are identical, i.e., $k_p = k_c$ and $k_p^* = k_c^*$. Physically, the functions $C(T)$ and $C'(T)$ characterize activation energies for the processes of surmounting, by dislocations, the barriers to their movement and of the climb of dislocations to parallel slip planes, respectively [5]. As a first approximation it is possible to regard these energies as equal and to set $C'(T) = k_c C(T)$. The functions $B(T)$ and $B'(T)$ are related to relevant activation volumes and as a first approximation it is also possible to assume $B'(T) = B(T)$.

With the above simplifications for selecting parameters of the model of a polycrystalline structural material, it is sufficient to have tensile diagrams for specimens at various temperatures and creep curves at various tensile stresses and temperatures; and for separation of the effects of isotropic and anisotropic hardening, it is sufficient to have experimental data for alternating-sign cyclic loading of the specimens. Parameters of the function f^{**} can be selected from the data on the rate of crystallization in annealing and the delay time for a variation in the yield strength under nonisothermal conditions. For cyclically stable materials, which do not display the properties of isotropic hardening or softening ($k_p^* = k_c^* = 0$ and $f^* \equiv 0$), under the assumptions made it is necessary to select only the functions k_p, k_T, k_T^*, and $C(T)$, $B(T)$.

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NOTATION

$T(x_1, x_2, x_3, t)$, absolute temperature; x_i $(i = 1, 2, 3)$, t, Cartesian coordinates and time; $\chi(x_1, x_2, x_3, t)$, thermodynamic temperature (scalar internal parameter); $\chi_i^{(1)}$, $\chi_i^{(2)}$, and χ_{ij}, internal parameters characterizing two mechanisms of heat propagation and nonequilibrium mechanical processes; τ_T, $\tau_q^{(1)}$, $\tau_q^{(2)}$, τ_σ, $\bar{\chi}$, $\bar{\chi}_i^{(1)}$, $\bar{\chi}_i^{(2)}$, $\bar{\chi}_{ij}$, relaxation times and steady-state values of relevant structural parameters; ρ, density; A, mass density of internal energy; $\varepsilon_{ij}^{(x)}$, $\varepsilon_{ij}^{(T)}$, components of deformation tensors determined by thermodynamic and absolute temperatures; $B_0(T)$, $B_1(T, \chi)$, functions defining variation in free energy solely as a result of variation in T and χ; T_0, temperature of natural state; σ_{ij}, components of stress tensor; h, mass density of energy; C_{ijkl}, D_{ijkl}, H_{ijkl}, components of symmetric tensor of elasticity coefficients and tensors specifying the effect of thermodynamic temperature and internal tensor parameter on stresses; $q_i(x_1, x_2, x_3, t)$, components of the vector of heat flux density; $\lambda_{ij}^{(1)}$, $\lambda_{ij}^{(2)}$, $\lambda_{ij}^{(3)}$, $\lambda_{ij}^{(4)}$, heat conduction tensors due to various mechanisms of heat propagation; $c_\varepsilon = -T\partial^2 B_0/\partial T^2$, $c'_\varepsilon = -T\partial^2 B_1/(\partial T/\partial\chi)$, specific mass heats at constant deformations due to absolute and thermodynamic temperatures; n_j, components of unit vector of outward normal to the surface of the front of heat wave propagation; $(\dot{\ }) = \partial(\ )/\partial t$; $\varepsilon^{(\mathrm{p})}$, plastic deformation; $\varepsilon^{(\mathrm{c})}$, creep; $\varepsilon^{(\mathrm{e})}$, elastic deformation; $\varepsilon^{(T)}$, thermal deformation; σ, σ', stress applied to the material specimen and mean value of microstresses in material; σ^*, yield strength of material; σ_0^*, yield strength of material after annealing; f', f^*, functions characterizing anisotropic and isotropic hardening of material, respectively; q_p^*, absolute accumulated plastic deformation; f_c, function defining the creep rate; f'', function specifying the rate of removal of anisotropic hardening of the material as a result of relaxation of microstresses; $E(T)$, elastic modulus of the material under tension; f^{**}, function characterizing the rate of removal of isotropic hardening; q_c^*, absolute accumulated creep; $C(T)$, $C'(T)$, $C^*(T)$ and $B(T)$, $B'(T)$, $B^*(T)$, temperature functions selected from results of testing the material specimens at different but fixed temperatures.

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